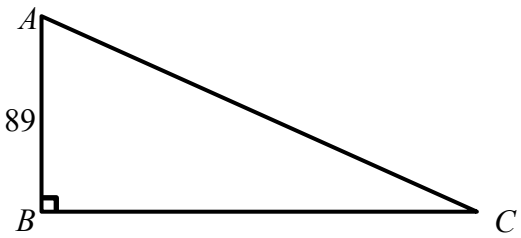
Hong Kong Mathematics Olympiad (2023/24)
Finals (Group – Event 1)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus Score			Time		
			Total Score				Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特别声明，答案须用数字表达，并化至最简。

1. Find the perimeter P of the right-angled triangle ABC if all the side lengths are positive integers and $AB = 89$.
若直角三角形 ABC 所有边长为正整数，且 $AB = 89$，求三角形 ABC 的周界 P。



2. If A is the units digit of $8888^{20242024}$, find the value of A.
若 A 是 的个位数 $8888^{20242024}$，求 A 的值。

3. How many 5-digit numbers contain at least one "1" and at least one "3"?
有多少个 5 位数包含最少 1 个「1」和最少 1 个「3」？

4. Let m be the number of possible pairs of positive integers a and b for which $a^4 + 4b^4$ is a prime number. Find m .
设有 m 对正整数 a 和 b，使 $a^4 + 4b^4$ 为质数，求 m 的值。

Hong Kong Mathematics Olympiad (2023/24)
Finals (Group – Event 2)

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1. Let $x > 0$. Given that $x - \frac{1}{x} = \sqrt{3}$ and $a = x^5 + x^3 + x + \frac{1}{x} + \frac{1}{x^3} + \frac{1}{x^5}$, find the value of a .

设 $x > 0$。已知 $x - \frac{1}{x} = \sqrt{3}$ 且 $a = x^5 + x^3 + x + \frac{1}{x} + \frac{1}{x^3} + \frac{1}{x^5}$ ，求 a 的值。

$a =$

2. Using the first 2024 positive integers: 1, 2, 3, 4, 5, 6, ... , 2024, a new integer is formed as 123456789101112...2024. If b is the number of "0" in this integer, find the value of b .

用首 2024 个正整数：$1, 2, 3, 4, 5, 6, \ldots, 2024$,造出一个新的整数：123456789101112...2024。若 b 是这个整数里「0」的数量，求 b 的值。

$b =$

3. c is the number of positive factors of $2024^2 - 2023^2$. Find the value of c .

c 是 $2024^2 - 2023^2$ 的正因子的数量。求 c 的值。

$c =$

4. Let "0", "1", "2", ……, "6" represent Sunday, Monday, Tuesday, …… and Saturday, respectively. Today is Monday. If "d " represents the day of week that comes after $20^{24^{2024}}$ days. Find d .

假设 「0」、「1」、「2」、……、「6」 分别为星期日、星期一 、星期二、……和星期六。今日是星期一，若 $20^{24^{2024}}$ 天后的那一天是星期几之代号为「d」，求 d。

$d =$

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1. Find the smallest positive integer n such that $2^{10}+2^{13}+2^n$ is a perfect square number.
试找出最小的正整数 n 使得 $2^{10}+2^{13}+2^n$ 成为一完全平方数。

2. Suppose $a^2+b^2+6a-14b+58=0$. Find $b-a$.
设 $a^2+b^2+6a-14b+58=0$。 求 $b-a$。

3. There was a chest containing \$8,000 buried in one of the corners of a square piece of land. In a contest, you and another man called "Mr. Badluck" were digging for the chest. Mr. Badluck had one peculiarity: he always made the wrong choice. You won the toss and chose first. You picked a corner, and Mr. Badluck picked another. Before you started, you observed that Mr. Badluck found no chest. The rules of the game allowed you to make a switch to another corner, but with a penalty of \$200. Should you make a switch? Calculate the expected gain from making the switch in dollars.
在正方形土地的某一个角落里埋着一个装有 \$8,000 的箱子。在一次比赛中，你和另一个叫「倒霉先生」的人一起挖箱子。倒霉先生有一个特点：他总是做出错误的选择。你赢了掷骰子先选。你选了一个角落，倒霉先生选了另一个角落。在你准备开始时，你发现倒霉先生没有找到箱子。游戏规则允许你换另一个角落，但要罚 \$200。你应否更换吗？计算换角落的期望收益。

4. A convex hexagon has the following property:

(i) all the triangles formed from any vertex with the two adjacent vertices have an area of 1,000 cm^2 ; and

(ii) $CH = DI$.

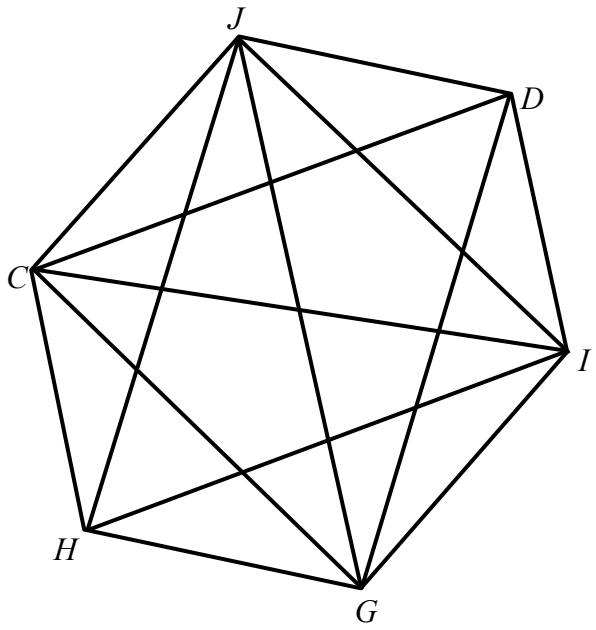
What is the area of the hexagon?

一个凸六边形有以下性质：

(i) 由任意顶点与相邻两个顶点组成的三角形的面积都是 1,000 cm^2；及

(ii) $CH = DI$ 。

求六边形的面积。



Hong Kong Mathematics Olympiad (2023/24)
Finals (Group – Event 4)

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1. Let a , b be non-zero integers satisfying the equation $a-ab+b=18$. Find $a+b$.
设 a、b 为非零整数，且满足方程 $a-ab+b=18$。求 $a+b$。

$a+b=$

2. Let x be a positive integer satisfying $x(x+1)(x+2)(x+3)=3024$. Find x .
设 x 为一正整数，且满足 $x(x+1)(x+2)(x+3)=3024$。求 x。

$x=$

3. Let α , β be the two roots of the quadratic equation $x^2+6x+2=0$. Find the quadratic equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$, and coefficient of x^2 is 1 .
设 α、β 为二次方程 $x^2+6x+2=0$ 的两个根，求以 $\frac{\alpha^2}{\beta}$ 和 $\frac{\beta^2}{\alpha}$ 为根及 x^2 的系数为 1 的二次方程。

4. The unshaded part in the diagram below is made up of a quarter-circle and a semi-circle. Find the area of the shaded part.
下图空白部分由一个四分一圆和一个半圆组成，求阴影部分的面积。